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Heat convection from a sphere placed in an oscillating free stream

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Abstract

The problem of heat convection from a sphere placed in an oscillating viscous free stream is considered for the two cases of forced and mixed convection regimes. The sphere surface is assumed isothermal and the free-stream oscillations are always in the vertical direction resulting in axisymmetric flow and thermal fields for both regimes. The study is based on the solution of the unsteady Navier–Stokes and energy equations for a Boussinesq fluid of constant Prandtl number (Pr = 0.71). The main parameters considered are the Reynolds number, Grashof number, and Strouhal number. Comparison of results with published data shows a good agreement for the special case of steady forced and mixed convection. The details of the resulting thermal fields are presented in the form of isotherm patterns and the local Nusselt number distribution.

Nomenclature

a radius of the sphere
f_n functions defined in eq. (7)
g gravitational acceleration
g_n functions defined in eq. (8)
Gr Grashof number, $g\beta(T_s - T_\infty)(2a)^3/v^2$
h_n functions defined in eq. (9)
N_u local Nusselt number
$\overline{N}_u$ overall Nusselt number
$\bar{\bar{N}}$ time-averaged Nusselt number
P_n Legendre polynomials of order n
P_n^1 first associated Legendre polynomials of order n
Pe Peclet number, Re Pr
Pr Prandtl number, v/α
r radial distance
Re Reynolds number, $2aU_o\, v$
S Strouhal number, $a\omega/U_o$
t dimensionless time
T temperature
U_o amplitude of oscillation of the free-stream velocity
U' free-stream velocity
z modified angular coordinate, $\cos\theta$.

Greek symbols

α thermal diffusivity
β coefficient of volumetric thermal expansion
δ Kronecker delta
ζ dimensionless vorticity
τ modified dimensionless time, $St/2\pi$
ξ logarithmic radial coordinate, $\xi = \ln(r/a)$
θ angular coordinate
v kinematic viscosity
φ dimensionless temperature
ψ dimensionless stream function
ω frequency of oscillations.

Subscripts

s at the surface of the sphere
∞ at infinite distance from the sphere

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1. Introduction

Heat transfer from a sphere has been the subject of many investigations due to the related engineering applications. Experimental and theoretical studies of natural convection from a sphere are numerous. The classic references include those of Potter and Riley [1], Geoola and Cornish [2] and [3], Riley [4], Brown and Simpson [5], Singh and Hasan [6], and Dudek et al. [7]. Forced convection from a sphere placed in a steady uniform stream was studied by Dennis and Walker [8] who investigated the phenomenon for Reynolds number up to 200 and Prandtl number up to 32 768. Another forced convection study was undertaken by Whitaker [9] who determined heat transfer correlations for flow past spheres. More accurate and detailed results were given by Dennis et al. [10] but for Reynolds number up to 20. Sayegh and Gauvin [11] solved the same problem and also investigated the effects of large temperature differences on the heat transfer rate. The mixed convection problem was studied by Hieber and Gebhart [12] who linearized the governing equations according to the matched asymptotic expansions of the perturbation theory and obtained solutions valid at small Reynolds and Grashof numbers. The problem was also solved using the boundary layer approach by Acrivos [13]. The work by Wong et al. [14] was one of the early attempts to solve the full steady Navier–Stokes and energy equations by the finite element method. Nguyen et al. [15] extended the work of Wong et al. [14] and solved the transient problem with internal thermal resistance.

The literature related to oscillating flow over a sphere is very rich. The problem dates back to the theoretical study of Basset [16] in 1888. Odar and Hamilton [17] conducted an experimental study on oscillating flow over a sphere in an attempt to modify the Basset solution of the drag by introducing coefficients for the drag components that were determined experimentally. Several attempts have since been made to decompose the drag force on a sphere based on its physical origin. According to Mei [18], the total unsteady force on a sphere consists of a quasi-steady force, a history force, an added-mass force, and the force due to unsteady free-stream acceleration. The form of the history force obtained by Mei [18] performed consistently better than that of Odar and Hamilton [17]. Other studies along these lines include those of Lawrence and Mei [19], Sano [20], Lovalenti and Brady [21], and Mei et al. [22]. Using perturbation methods, Riley [23] investigated the flow induced by a sphere oscillating at high frequency in a viscous fluid when the amplitude of oscillation is small compared with the radius of the sphere. He concluded that the condition $Re_s \gg 1$ ($Re_s = Re/S$) was a necessary and sufficient condition for a high-frequency flow to have a double boundary layer structure. This was not the case for the low frequency flows investigated by Chang and Maxey [24] where flows with $Re_s \geqslant O(1)$ were the only cases exhibiting double boundary layer structures. The study by Chang and Maxey [24], while it represents a comprehensive treatment of oscillatory flow over spheres, is limited to the low Reynolds number range. Recently, Alassar and Badr [25] extended the work of Chang and Maxey [24] to Reynolds number of 200 and included a detailed analysis of the separation angle and the wake length.

Studies related to heat or mass transfer from a sphere in an oscillating free stream are scarce. Perhaps, the most important are those by Drummond and Lyman [26], and Ha and Yavuzkurt [27]. These two studies, however, deal with forced convection only. Drummond and Lyman [26] studied mass transfer from a sphere in an oscillating flow using a pseudo-spectral method. It seems that the flow field did not reach a quasi-steady state since the streamline pattern was not periodic. It was concluded that the mass transfer rate decreases with the decrease of the Strouhal number until reaching $S = 2$ below which the rate is virtually independent of the Strouhal number. As stated by Drummond and Lyman [26], this last conclusion contradicted other experimental and theoretical studies. A good study on heat transfer from a sphere in an oscillating free stream was carried out by Ha and Yavuzkurt [27]. The study, however, deals with only forced convection.

The present paper deals with forced and mixed convection heat transfer from a sphere in an axisymmetric oscillating flow. The oscillations are harmonic and the flow is governed by the Navier–Stokes equations for incompressible fluids. The parameters involved in the problem are the Reynolds number, Strouhal number, and Grashof number. The Prandtl number is fixed at a value of 0.71. The method of solution adopted here is the series truncation method where the stream function, temperature, and vorticity are expanded in terms of Legendre and first associated Legendre functions. A directly related application of this work is in the use of an acoustic field for enhancing heat transfer from a sphere. An experimental study of this aspect was carried out by Leung and Baroth [28] using holographic interferometry for flow visualization. They reported that the presence of an acoustic field enhances heat transfer when the vibrational Reynolds number exceeds 400.

2. Basic equations and method of solution

The problem considered is that of mixed (forced and natural) convection from an isothermal sphere placed in an oscillating free stream of infinite extent as shown in Fig. 1. The free-stream oscillations are in the vertical direction resulting in an axisymmetric flow and thermal fields. The equations governing the heat transfer process, cast in spherical coordinates, can be written in terms of

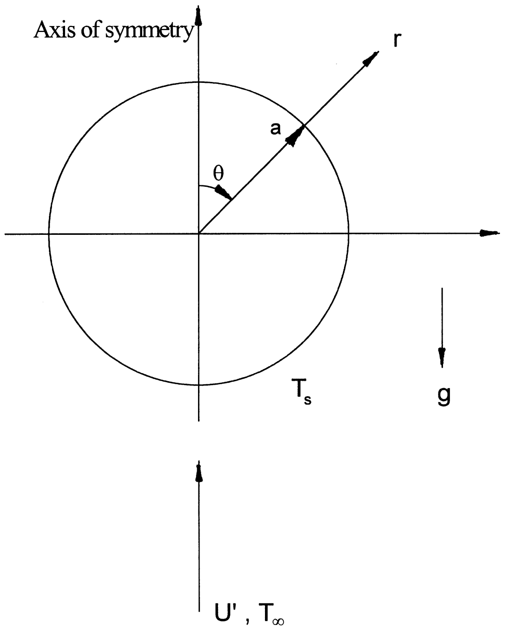

Fig. 1. The spherical coordinate system.

the dimensionless vorticity, ζ, the dimensionless stream function, ψ, and the dimensionless temperature, φ, as

$$e^{3\xi}\sin\theta\zeta+\frac{\partial^2\psi}{\partial\xi^2}+\frac{\partial^2\psi}{\partial\theta^2}-\frac{\partial\psi}{\partial\xi}-\cot\theta\frac{\partial\psi}{\partial\theta}=0 \quad (1)$$

$$e^{2\xi}\frac{\partial\zeta}{\partial t}+\frac{e^{-\xi}}{\sin\theta}\left[\frac{\partial\psi}{\partial\theta}\left(\frac{\partial\zeta}{\partial\xi}-\zeta\right)-\frac{\partial\psi}{\partial\xi}\left(\frac{\partial\zeta}{\partial\theta}-\cot\theta\zeta\right)\right]$$
$$=\frac{2}{\mathrm{Re}}\left(\frac{\partial^2\zeta}{\partial\xi^2}+\frac{\partial^2\zeta}{\partial\theta^2}+\frac{\partial\zeta}{\partial\xi}+\cot\theta\frac{\partial\zeta}{\partial\theta}-\frac{\zeta}{\sin^2\theta}\right)$$
$$-e^{\xi}\frac{\mathrm{Gr}}{2\mathrm{Re}^2}\left(\sin\theta\frac{\partial\varphi}{\partial\xi}+\cos\theta\frac{\partial\varphi}{\partial\theta}\right) \quad (2)$$

$$e^{2\xi}\frac{\partial\varphi}{\partial t}+\frac{e^{-\xi}}{\sin\theta}\left[\frac{\partial\psi}{\partial\theta}\frac{\partial\varphi}{\partial\xi}-\frac{\partial\psi}{\partial\xi}\frac{\partial\varphi}{\partial\theta}\right]$$
$$=\frac{2}{\mathrm{Pe}}\left[\frac{\partial^2\varphi}{\partial\xi^2}+\frac{\partial^2\varphi}{\partial\theta^2}+\frac{\partial\varphi}{\partial\xi}+\cot\theta\frac{\partial\varphi}{\partial\theta}\right] \quad (3)$$

where $\mathrm{Re}=2aU_o/\nu$ is the Reynolds number, $\mathrm{Gr}=g\beta(T_s-T_\infty)(2a)^3/\nu^2$ is the Grashof number, $\mathrm{Pe}=\mathrm{Re}\,\mathrm{Pr}$ is the Peclet number, $\mathrm{Pr}=\nu/\alpha$ is the Prandtl number, a is the radius of the sphere, U_o is the amplitude of oscillation of the free-stream velocity, ν is the coefficient of kinematic viscosity, g is the gravitational acceleration, t is the time, β is the coefficient of volumetric thermal expansion, and α is the thermal diffusivity. The logarithmic transformation $\xi=\ln(r/a)$ is used, where r is the dimensional radial distance. The variables ψ, ζ, φ, and t in the governing equations are defined in terms of the usual dimensional quantities ψ', ζ', T, and t' as: $\psi=\psi'/U_oa^2$, $\zeta=\zeta'a/U_o$, $\varphi=(T-T_\infty)/(T_s-T_\infty)$, and $t=U_ot'/a$, where T_∞ is the free-stream temperature, T_s is the temperature at the surface of the sphere. The oscillations in the free-stream velocity are given in the form $U=U'/U_o=\cos(\mathrm{S}t)$ where U' is the dimensional free-stream velocity and $\mathrm{S}=a\omega/U_o$ is the Strouhal number with ω the frequency of oscillations.

The boundary conditions are mainly the no-slip, impermeability and isothermal conditions on the sphere surface and the uniform velocity and temperature far away from it. These can be expressed as

$$\psi=\frac{\partial\psi}{\partial\theta}=\frac{\partial\psi}{\partial\xi}=0,\quad \text{and}\ \varphi=1\quad \text{at}\ \xi=0 \quad (4)$$

$$\left.\begin{aligned}&\frac{\partial\psi}{\partial\xi}\approx e^{2\xi}\sin^2\theta\cos(\mathrm{S}t),\\ &\text{and}\quad \frac{\partial\psi}{\partial\theta}\approx e^{2\xi}\sin\theta\cos\theta\cos(\mathrm{S}t)\\ &\text{or}\quad \psi\approx\frac{e^{2\xi}}{2}\sin^2\theta\cos(\mathrm{S}t)\end{aligned}\right\}\text{as}\ \xi\to\infty \quad (5)$$

and

$$\varphi\to 0,\quad \text{and}\ \zeta\to 0\quad \text{as}\ \xi\to\infty \quad (6)$$

The method of solution is based on approximating ψ, ζ, and φ using Legendre and first associated Legendre polynomials which can be written in the following form:

$$\psi=e^{\xi/2}\sum_{n=1}^{\infty}f_n(\xi,t)\times\int_z^1 P_n(\gamma)\,d\gamma \quad (7)$$

$$\zeta=\sum_{n=1}^{\infty}g_n(\xi,t)P_n^1(z) \quad (8)$$

$$\varphi=\sum_{n=0}^{\infty}h_n(\xi,t)P_n(z) \quad (9)$$

where $P_n(z)$, and $P_n^1(z)$ are respectively the Legendre and first associated Legendre polynomials of order n, and $z=\cos\theta$. Using these expansions in eqs. (1)–(3), the following differential equations for Legendre coefficients are obtained:

$$\frac{\partial^2 f_n}{\partial\xi^2}-(n+1/2)^2f_n=n(n+1)e^{5/2\xi}g_n \quad (10)$$

$$e^{2\xi}\frac{\partial g_n}{\partial t}=\frac{2}{\mathrm{Re}}\left(\frac{\partial^2 g_n}{\partial\xi^2}+\frac{\partial g_n}{\partial\xi}-n(n+1)g_n\right)$$
$$-e^{\xi}\frac{\mathrm{Gr}}{2\mathrm{Re}^2}\left[\frac{1}{(2n-1)}\left((n-1)h_{n-1}-\frac{\partial h_{n-1}}{\partial\xi}\right)\right.$$
$$\left.+\frac{1}{(2n+3)}\left((n+2)h_{n+1}+\frac{\partial h_{n+1}}{\partial\xi}\right)\right]+S_n \quad (11)$$

$$e^{2\xi}\frac{\partial h_n}{\partial t}=\frac{2}{\mathrm{Pe}}\left(\frac{\partial^2 h_n}{\partial \xi^2}+\frac{\partial h_n}{\partial \xi}-n(n+1)h_n\right)+H_n \tag{12}$$

where,

$$S_n=-e^{-\xi/2}\left[\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\alpha_{ij}^n f_i\left(\frac{\partial g_j}{\partial \xi}-g_j\right)+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\beta_{ij}^n g_j\left(\frac{\partial f_i}{\partial \xi}+\frac{1}{2}f_i\right)\right] \tag{13}$$

$$H_n=-e^{-\xi/2}\left[\sum_{i=1}^{\infty}\sum_{j=0}^{\infty}\frac{2n+1}{2i+1}\alpha_{nj}^i h_j\left(\frac{\partial f_i}{\partial \xi}+\frac{1}{2}f_i\right)+\sum_{i=1}^{\infty}\sum_{j=0}^{\infty}\gamma_{ij}^n f_i\frac{\partial h_j}{\partial \xi}\right] \tag{14}$$

The coefficients appearing in the series are defined in terms of the $3j$ symbols

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

as follows:

$$\alpha_{ij}^n=-(2n+1)\sqrt{\frac{j(j+1)}{n(n+1)}}\begin{pmatrix} n & i & j \\ -1 & 0 & 1 \end{pmatrix}\times\begin{pmatrix} n & i & j \\ 0 & 0 & 0 \end{pmatrix} \tag{15}$$

$$\beta_{ij}^n=(2n+1)\sqrt{\frac{j(j^2-1)(j+2)}{n(n+1)i(i+1)}}\begin{pmatrix} n & i & j \\ -1 & -1 & 2 \end{pmatrix}\times\begin{pmatrix} n & i & j \\ 0 & 0 & 0 \end{pmatrix} \tag{16}$$

and

$$\gamma_{ij}^n=(2n+1)\begin{pmatrix} n & i & j \\ 0 & 0 & 0 \end{pmatrix}^2 \tag{17}$$

The boundary conditions associated with eqs. (10)–(12) are

$$f_n(0,t)=\frac{\partial f_n}{\partial \xi}(0,t)=0, \quad \text{and } h_n(0,t)=\delta_{n0} \tag{18}$$

$$f_n(\xi,t)\approx e^{3/2\xi}\cos(\mathrm{S}t)\delta_{n1},$$
$$\frac{\partial f_n(\xi,t)}{\partial \xi}\approx\frac{3}{2}e^{3/2\xi}\cos(\mathrm{S}t)\delta_{n1} \text{ as } \xi\to\infty \tag{19}$$

and

$$g_n(\xi,t)\to 0, \quad \text{and } h_n(\xi,t)\to 0 \text{ as } \xi\to\infty \tag{20}$$

where δ is the Kronecker delta. Finally, an integral condition that must be satisfied by the functions g_n can be obtained from eq. (10) by making use of the boundary conditions eqs. (18) and (19):

$$\int_0^\infty e^{(2-n)\xi}g_n\,d\xi=\frac{3}{2}\cos(\mathrm{S}t)\delta_{n1} \tag{21}$$

The potential flow solution is used as an initial solution (at $t=0$) at the start of the motion. Although this is not a perfect assumption, the effect of this initial condition becomes insignificant at large time when quasi-steady state is reached. The solutions of the functions ψ and ς are advanced in time by solving eq. (11) using a Crank–Nicolson finite-difference scheme and solving eq. (10) using a specialized step-by-step technique. The numerical method is discussed at length by Alassar and Badr [25] and need not be repeated. The method is extended here for the energy eq. (12) which is strongly coupled with the flow equations. Moreover, the dimensionless time τ which is related to the previously defined dimensionless time t by $\tau=\mathrm{S}t/2\pi$ is introduced in the present study. Scaling time by the Strouhal number is appropriate in dealing with relatively high frequency flows. Consequently, each cycle has a period of unity with 400 divisions which makes $\Delta\tau=0.0025$.

The local and overall Nusselt numbers, N_u and $\overline{N}_u$ are obtained from

$$N_u(\theta,\tau)=-2\left(\frac{\partial\varphi}{\partial\xi}\right)_{\xi=0}=-2\sum_{n=0}^{\infty}\frac{\partial h_n}{\partial\xi}P_n(\cos\theta) \tag{22a}$$

$$\overline{N}_u(\tau)=\frac{1}{2}\int_0^\pi N_u(\theta,\tau)\sin\theta\,d\theta=-2\frac{\partial h_0}{\partial\xi} \tag{22b}$$

Equations (22b) is based on averaging over the entire surface of the sphere. The time-averaged Nusselt number $\bar{\bar{N}}$ is obtained by averaging $\overline{N}_u(\tau)$ over a complete cycle and becomes constant when quasi-steady state is reached. The quantity $\bar{\bar{N}}_u(\tau)$ is obtained from

$$\bar{\bar{N}}_u(\tau)=\int_{\tau-1/2}^{\tau+1/2}\overline{N}_u(x)\,dx \tag{23}$$

The accuracy of the method of solution was verified by studying the case of forced convection from a sphere in a uniform free stream. The steady values of the overall Nusselt number were compared with previously known solutions. Table 1 shows a good agreement between the values of $\overline{N}_u$ obtained from the present study and those of Wong et al. [14], Sayegh and Gauvin [11], Dennis and Walker [10], Whitaker [9], and Dennis and Walker [8]. Another comparison is shown in Table 2 for the case of steady mixed convection where the results of the present study are within 5% of those of Wong et al. [14].

3. Results and discussion

The effect of free-stream oscillations on heat transfer from a small sphere is investigated for the two cases of forced and mixed convection regimes. Axisymmetry is maintained in mixed convection by allowing the free-stream oscillations only in the vertical direction. The forced convection problem is solved in the range of Reynolds number up to Re = 200 while the mixed convection

Table 1
Comparison between the overall Nusselt number obtained from the present study and those reported by other researchers for the special case of steady forced convection

Re	Present	Wong et al. [14]	Sayegh and Gauvin [11]	Dennis and Walker [10]	Whitaker [9]	Dennis and Walker [8]
1	2.263	–	2.232	2.260	2.402	2.22
10	3.326	3.282	3.323	3.358	3.349	3.18
20	4.046	3.971	4.022	4.065	3.950	3.86
30	4.584	–	4.560	–	4.421	4.36
60	5.855	5.889	–	–	5.511	5.65
100	7.153	7.351	–	–	6.625	7.11

Table 2
Comparison between the overall Nusselt number obtained from the present study and those obtained by Wong et al. [14] for steady mixed convection

Re	Gr	Present	Wong et al. [14]	Percentage difference
60	720	5.943	6.021	1.31
60	14,400	7.331	7.183	2.02
100	2000	7.190	7.509	4.44

problem is solved for Reynolds numbers up to Re = 20 and Grashof numbers up to Gr = 4000.

3.1. The forced convection regime

Due to the free-stream oscillations, the velocity and the thermal fields near the surface of the sphere change with time in a periodic fashion. Figure 2 shows the local Nusselt number distribution at different times over a complete cycle of oscillation for the case of Re = 50 and S = π/4. The distributions obtained at $\tau = 0$ and $\tau = 1$ are very much the same confirming the periodicity of the thermal field. The maximum value of Nusselt number, $N_{u\,max}$, occurs either at $\theta = 0$ or 180° depending on the direction of the free stream. However, this $N_{u\,max}$ does not occur at $\tau = 0$ or $\tau = 0.5$ (corresponding to the maximum free-stream velocity) but instead occurs nearly at $\tau = 0.125$ or $\tau = 0.625$. The phase shift may be attributed to the phase lag between the free-stream oscillations and the fluctuation of the momentum and thermal boundary layers in the immediate neighborhood of the sphere (for a good discussion on this issue, see Ha and Yavuzkurt [27]). The viscous forces in the boundary layer region are added to the inertia forces during the acceleration phase of motion which results in a phase lag between the boundary layer flow and the free-stream flow. This phenomenon was also reported in the work by Badr [29] for the case of fluctuating flow over a cylinder at low Reynolds and Strouhal numbers (Re = 50, S = π/4).

Figure 3 shows the time variation of N_u distribution over a complete cycle for the low frequency case of Re = 10, S = π/16. Contrary to the behavior shown in Fig. 2, $N_{u\,max}$ occurs nearly at times of maximum free-stream velocity ($\tau = 0$, 0.5, 1) which indicates a significant decrease in the phase lag between the boundary layer and the free stream. The time variation of the overall Nusselt number during one complete cycle for different Reynolds numbers when S = π/4 are plotted in Fig. 4. The corresponding time developments of $\bar{N}_u$ following the start of motion are shown in Fig. 5. It can be seen that the phase lag of the overall Nusselt number decreases with increasing Reynolds number. Two maximum values of $\bar{N}_u$ corresponding to the two peaks of the free-stream velocity and two minimum values of $\bar{N}_u$ corresponding to zero free-stream velocity during each complete cycle are clearly observed in Fig. 4. Table 3 lists the times (τ) at which the maximum and minimum values of $\bar{N}_u$ occur for the range of parameters considered in the present study.

Ha and Yavuzkurt [27] observed differences between the values of the overall Nusselt number corresponding to the maximum positive velocity and those corresponding to the maximum negative velocity and attributed this to acoustic streaming and the phase lag between the applied acoustic field and the thermal boundary layer. The same observation was reported by Tseng and Lin [30]. Table 4 shows that the values of the overall Nusselt number corresponding to the maximum negative

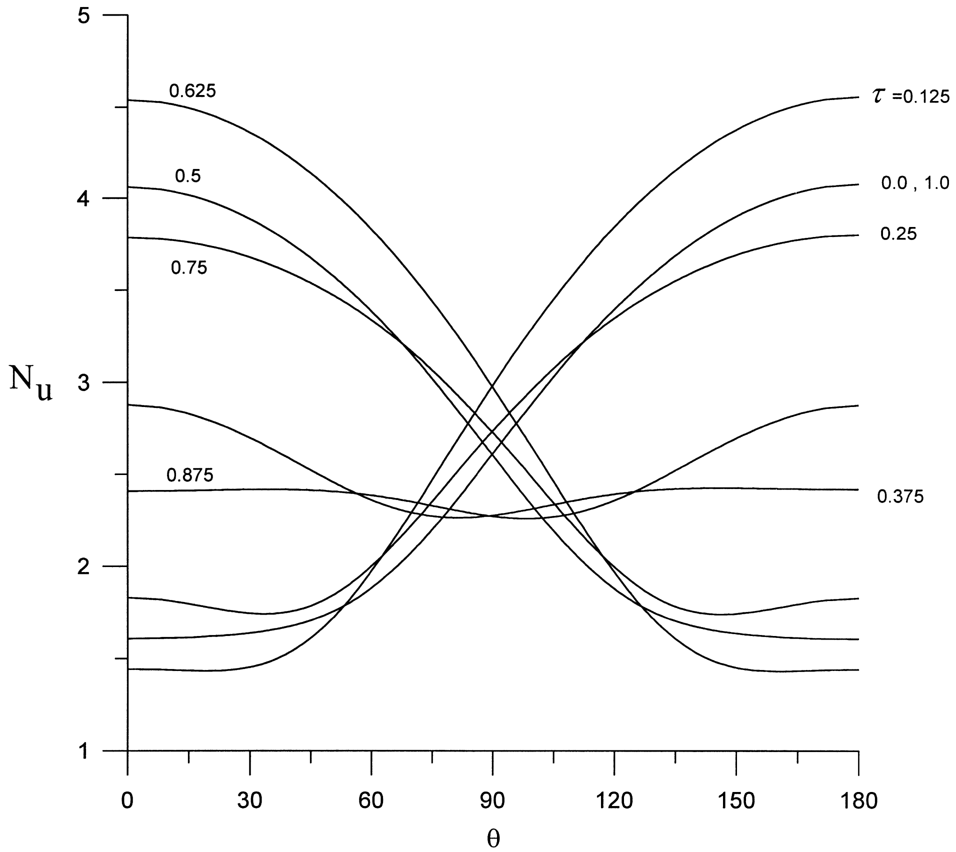

Fig. 2. The local Nusselt number distribution and its time variation during one complete cycle for the case of Re = 50, S = π/4, Gr = 0.

velocity are smaller than those corresponding to the maximum positive velocity. The difference between the peak values can not be attributed to steady streaming since the later is time independent (see [18, 24, 25]). The higher values of the overall Nusselt number at the maximum positive velocity can be explained by the fact that at small time following the start of motion, the thermal field is more developed at the upper half since the motion starts upward. The converse is expected if instead the free-stream velocity is a negative cosine function. The symmetry between the upper and lower halves of the thermal field is normally achieved at large time. Table 4 shows that the difference in $\overline{N}_u$ becomes negligibly small at cycle 20 for all cases considered. Table 4 also shows that $\overline{N}_u$ at maximum velocity attains a minimum between $S = \pi/16$ and π for the case of Re = 10.

To investigate the effect of Strouhal number on $\overline{N}_u$, the case of Re = 10 is studied for values of S between $\pi/16$ and 2π. Figure 6 shows the time variation of $\overline{N}_u$ for six values of S during cycle 320. The solution was advanced in time until reaching this cycle in order to achieve quasi-steady thermal field. The figure shows a continuous decrease of $\overline{N}_u$ (both average values and amplitude) with increasing S until reaching $S = \pi/2$. Further increase of S results in an increase in the average $\overline{N}_u$ but with less amplitude. The time development of $\bar{\bar{N}}_u$ following the start of motion for the case of Re = 10 and different values of S is shown in Fig. 7. It is clear from the figure that there is a critical value of S at which heat transfer is minimum. This phenomenon can also be observed in Fig. 8 which shows the effect of Strouhal number on $\bar{\bar{N}}_u$ after carrying the solution to different numbers of cycles.

The effect of steady streaming on heat transfer is examined by plotting the time-averaged stream function for the different Strouhal numbers considered at Re = 10 as shown in Fig. 9. It is worth noting that Riley [23] found that the condition $Re/S \gg 1$ is necessary and sufficient for a low-amplitude (high frequency) oscillatory flow to have a double-boundary layer structure. This was not the case for the high-amplitude (low frequency) flows considered by Chang and Maxey [24]. They found that the only flows exhibiting two streaming regions were those where $Re/S \geqslant O(1)$ with low- to moderate-amplitude oscillations. No generalizations, however, were made with respect to the conditions under which a double-boundary layer structure exists. Figure 9 shows that such a structure

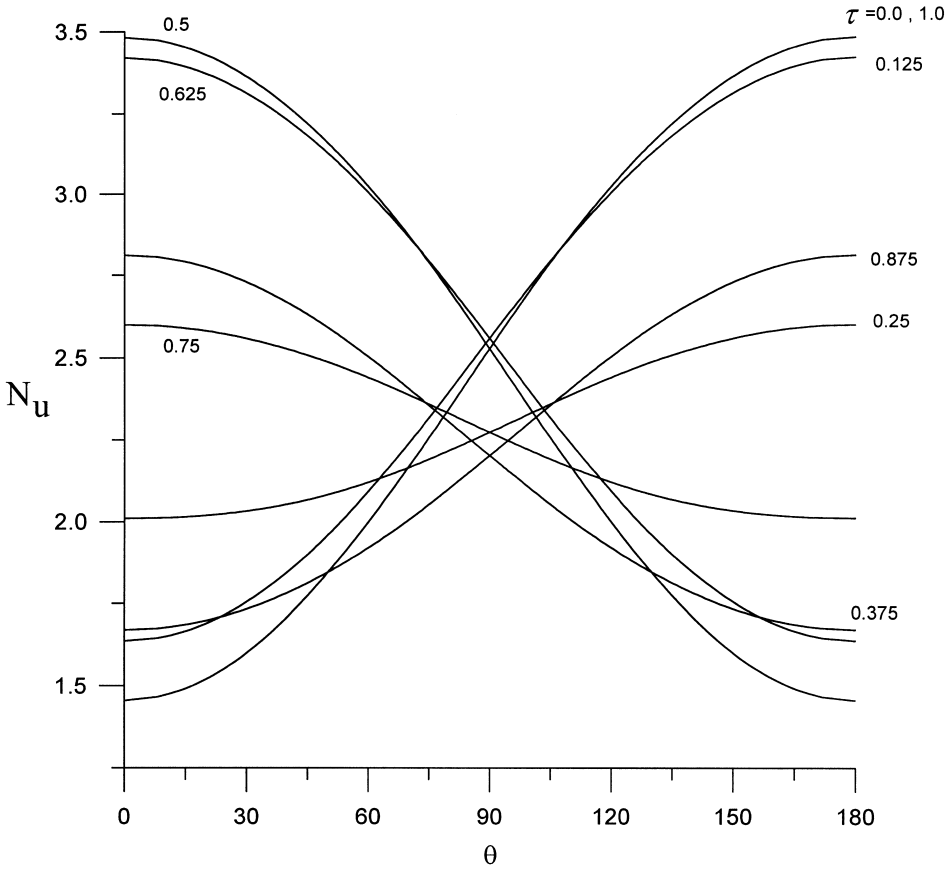

Fig. 3. The time variation of N_u distribution for the case of Re = 10, S = π/16, Gr = 0 during one complete cycle.

is responsible for the existence of a critical S value at which a minimum value of $\bar{\bar{N}}_u$ is observed. It is exactly when the double-boundary layer structure disappears (or extends very far in the main stream) that this minimum $\bar{\bar{N}}_u$ occurs. This conclusion was also checked against the case of Re = 5.

The isotherm patterns during one half of a complete oscillation for the case of Re = 200, and S = π/4 are shown in Fig. 10. At this Strouhal number, the amplitude of oscillation is relatively small and little changes in the temperature distribution of the far-field (the left parts of the figures) with time are observed as opposed to the near-field distribution (the right parts of the figures). Heat transfer by diffusion plays an important role far away from the surface of the sphere.

3.2. The mixed convection regime

The investigation of the mixed convection regime is carried for the case of axisymmetric flow when both free-stream oscillations and buoyancy driven flow are in the vertical direction. The study is carried out for two Reynolds numbers (Re = 5 and 20) and for Grashof numbers ranging from 0 to 10 Re^2 while keeping the Strouhal number unchanged (S = π/4). The effect of buoyancy driven flow on the velocity and thermal fields increases with the increase of the parameter Gr/Re^2. Figure 11 shows the time variation of the local Nusselt number distribution over a complete cycle for the case of Re = 5, S = π/4 and $Gr/Re^2 = 10$. The maximum value of N_u occurs always at $\theta = 180^\circ$ rather than oscillating between $\theta = 0$ and 180° as in the case of forced convection. In addition, $N_{u\,max}$ occurs at the lower stagnation point at $\tau = 0$ at which the free-stream velocity is maximum and in the same direction of the buoyancy driven flow while $N_{u\,min}$ at the same point occurs at $\tau = 0.5$ at which the two streams oppose each other. While the time variation of N_u at the lower stagnation point ($\theta = 180^\circ$) is considerable, the variation at the upper stagnation point ($\theta = 0$) is minimal. Another interesting phenomenon that can be

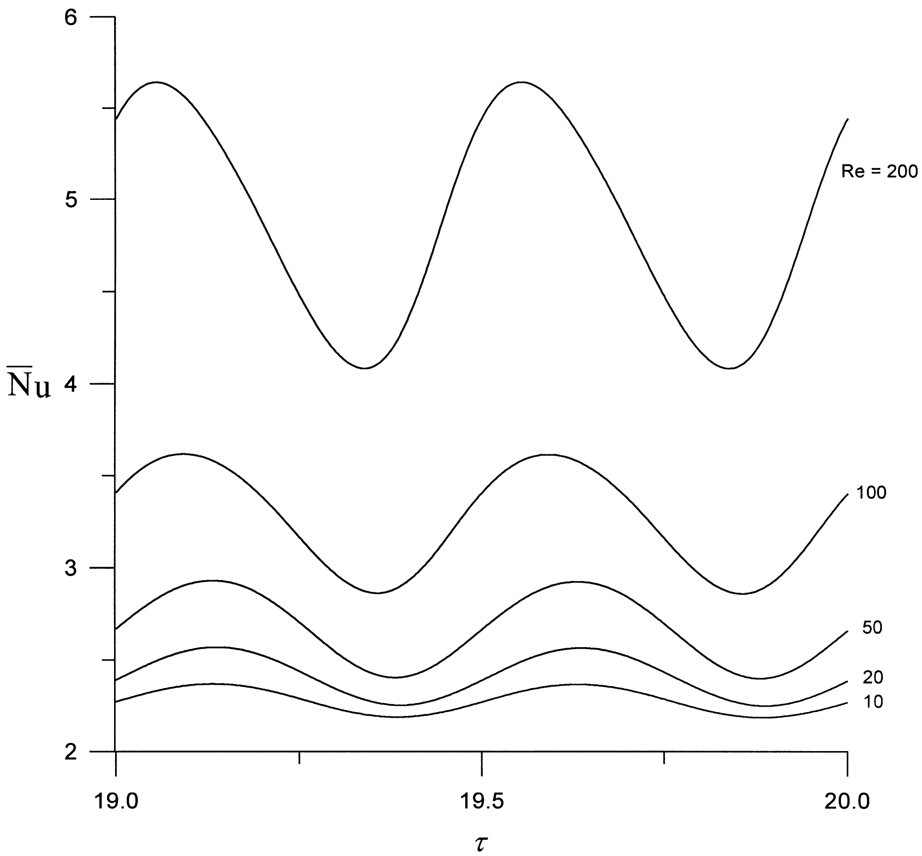

Fig. 4. The time variation of $\overline{N}_u$ over one complete cycle for the case of $S=\pi/4$, $Gr=0$ and different Reynolds numbers.

seen in Fig. 12 is that the frequency of oscillation of $\overline{N}_u$, in the presence of buoyancy driven flow, is one half of that for forced convection. That is mainly because the thermal field is no longer symmetric during the upward and downward motions of the free stream.

The overall effect of the free-stream oscillations on heat transfer in the presence of buoyancy driven flow is given in Table 5 for all cases considered. The table shows about 50% increase of $\bar{\bar{N}}_u$ at $Re=5$, $Gr/Re^2=10$ over that of forced convection. That increase reaches about 100% as Re increases to 20. It was reported by Wong et al. [14] and Nguyen et al. [15] who studied convection from a sphere in a uniform stream that motion induced by density variation only produces minor effects on the Nusselt number unless the ratio Gr/Re^2 is greater than 0.5. Their results showed that the value $Gr/Re^2=0.5$ induces a change of nearly 5% in the value of $\overline{N}_u$ from its corresponding forced convection value. However, Table 5 shows that the conclusion by Wong et al. [14] and Nguyen et al. [15] does not apply to the case of oscillating flow. For example, an increase of 25% in $\bar{\bar{N}}_u$ is observed when $Gr/Re^2=0.25$ at $Re=20$. This can be understood through the fact that, at the same Re, the Nusselt number for the steady forced convective heat transfer is larger than that for the oscillating forced one. Consequently, the relative change in the Nusselt number induced by natural convection is higher for the oscillating flow case.

4. Conclusions

The problem of heat convection from a heated sphere placed in an oscillating free stream is studied for the two cases of forced and mixed convection regimes. In the case of forced convection, the study revealed that the velocity and thermal fields are symmetric during the upward and downward motions of the free stream. The phase lag between the free-stream oscillations and the momentum and thermal boundary layers is found to decrease with the decrease of Strouhal number. The overall rate of heat transfer exhibits a minimum at a certain critical value of the Strouhal number at which the double boundary layer structure disappears. In the case of mixed convection, the

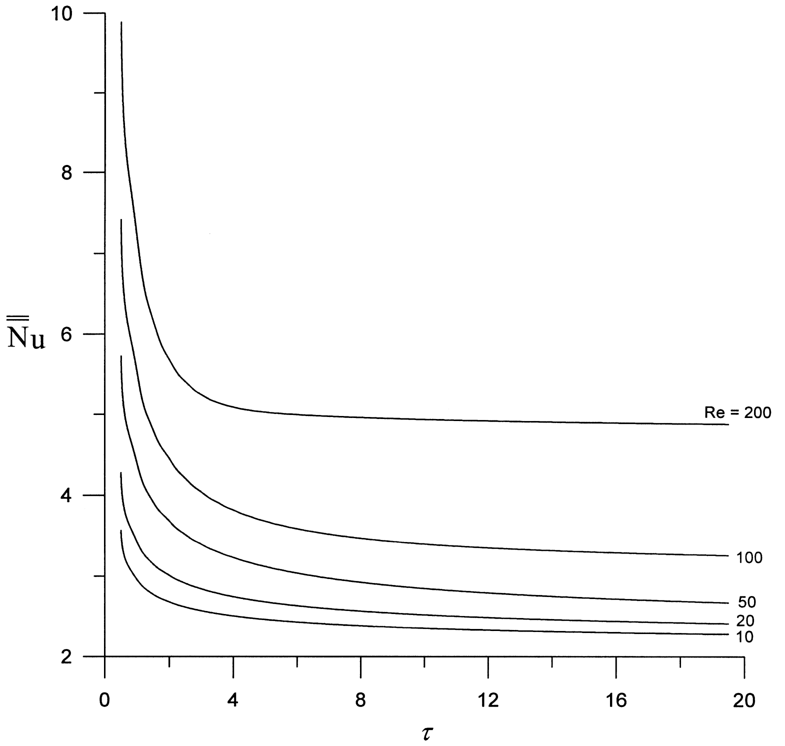

Fig. 5. The time variation of $\bar{\bar{N}}_u$ following the start of motion for the case of $S = \pi/4$, $Gr = 0$ and different Reynolds numbers.

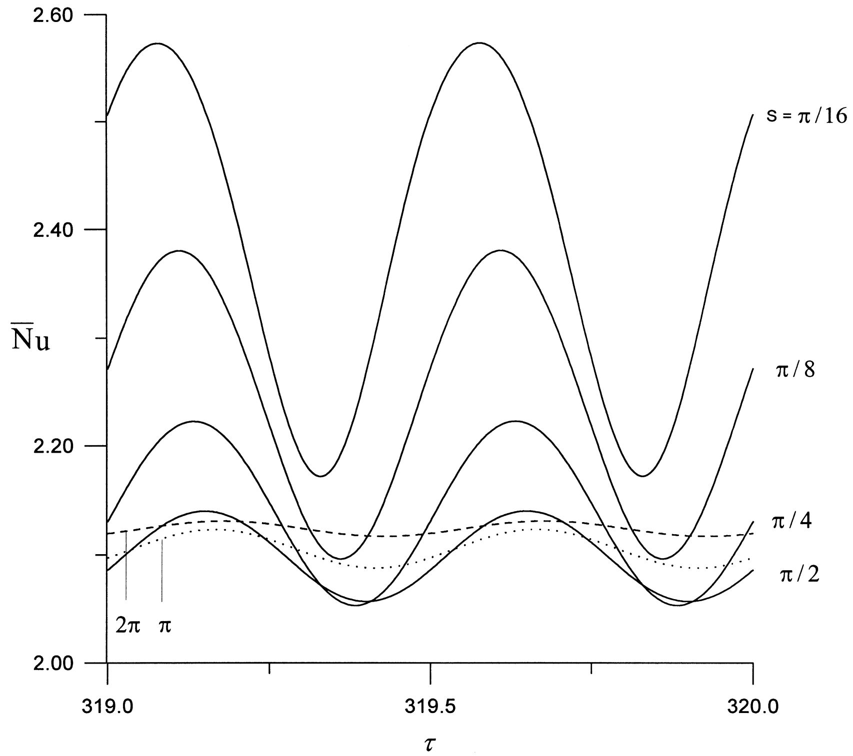

Fig. 6. The time variation of $\overline{N_u}$ for the case of $Re = 10$, $Gr = 0$ at different Strouhal numbers during cycle 320.

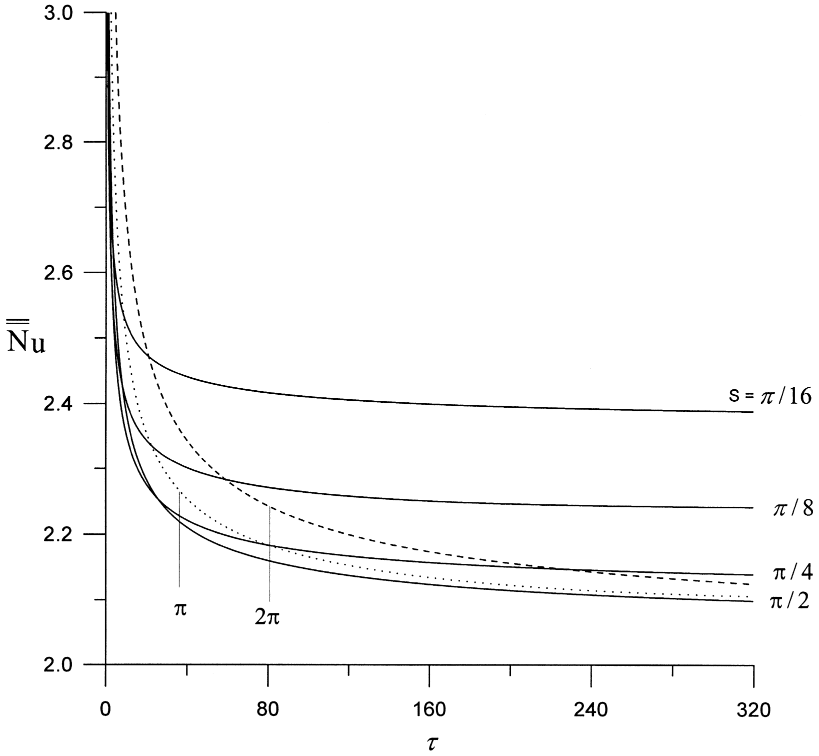

Fig. 7. The time variation of $\bar{\bar{N}}_u$ for the case of Re = 10 following the start of motion.

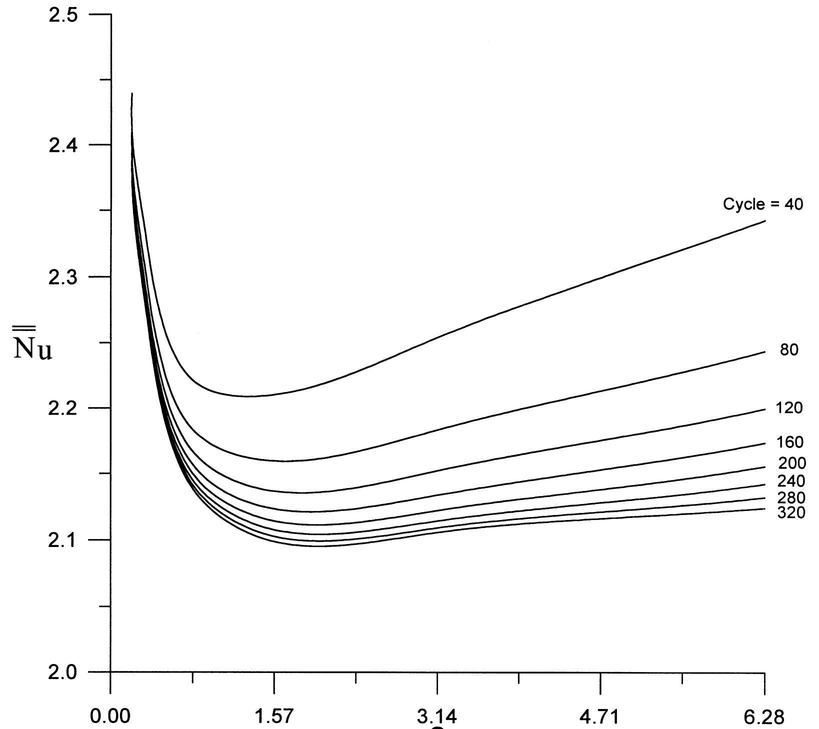

Fig. 8. The variation of $\bar{\bar{N}}_u$ with S for the case of Re = 10 at different cycles.

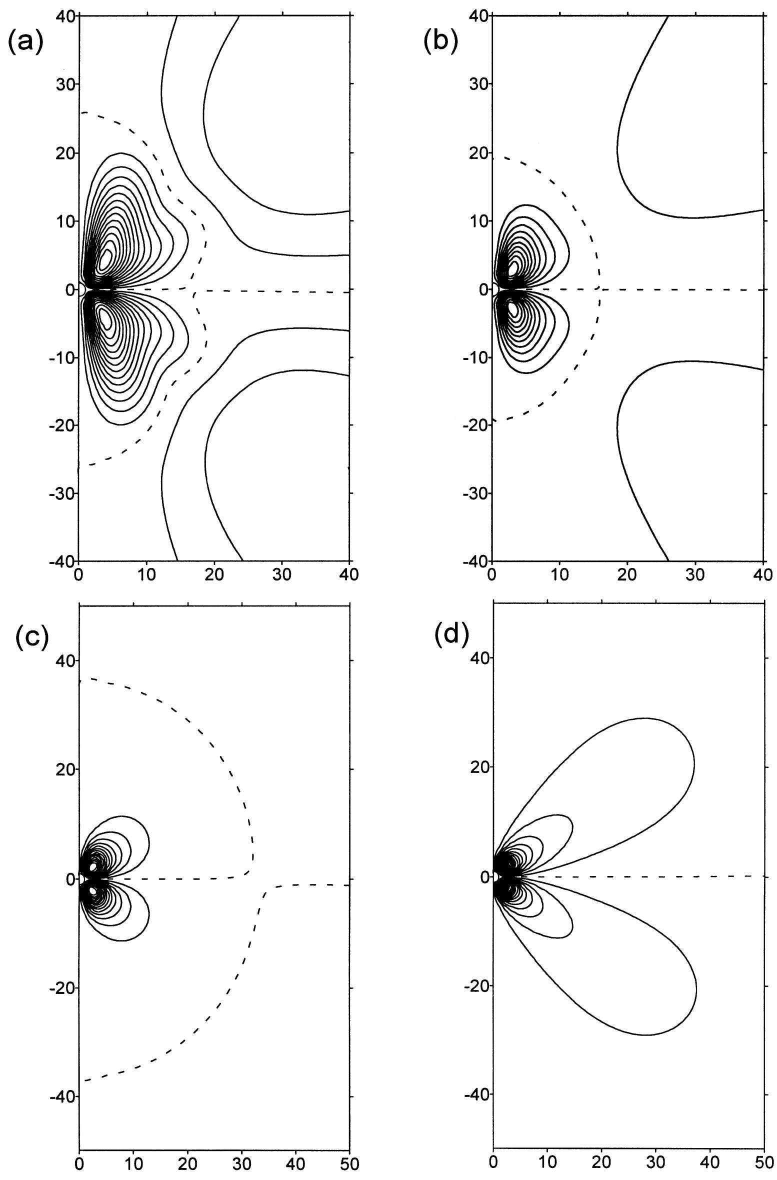

Fig. 9. Time averaged stream function for the case of Re = 10. (a) S = π/32, $\Delta\psi = 0.02$, (b) S = π/16, $\Delta\psi = 0.02$, (c) S = π/8, $\Delta\psi = 0.01$, (d) S = π/4, $\Delta\psi = 0.005$, (e) S = π/2, $\Delta\psi = 0.0025$, (f) S = π, $\Delta\psi = 0.001$, (g) S = 2π, $\Delta\psi = 0.001$.

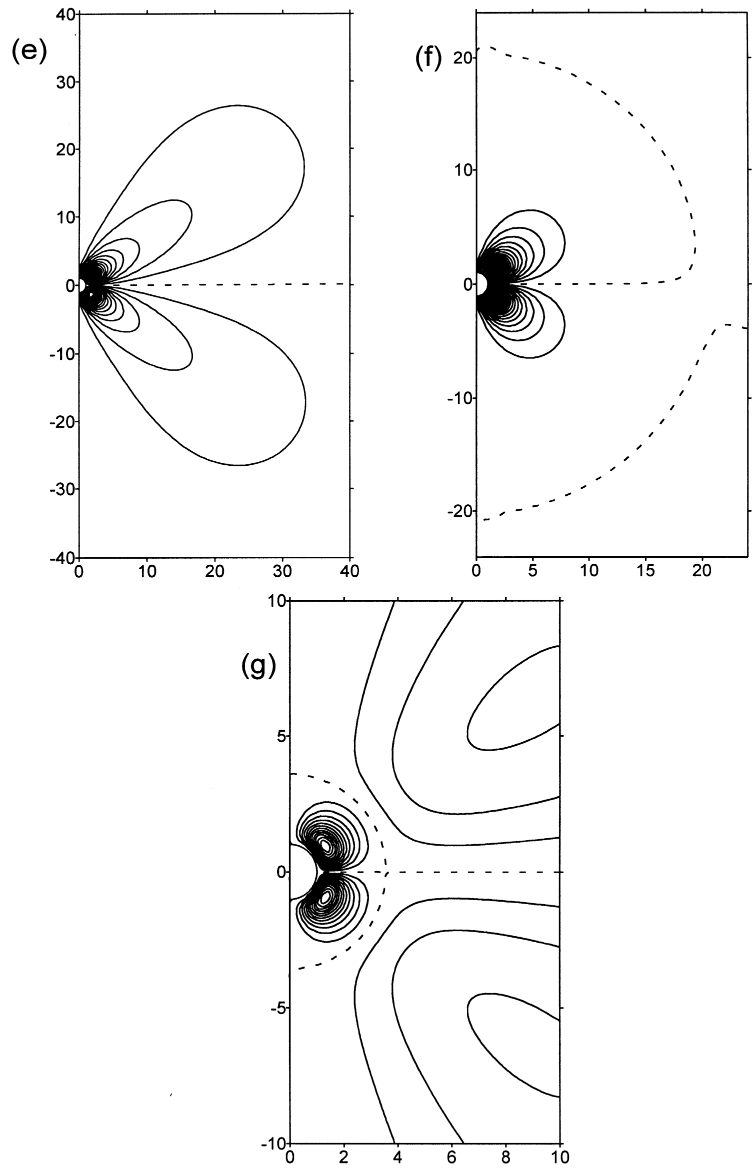

Fig. 9 (*continued*)

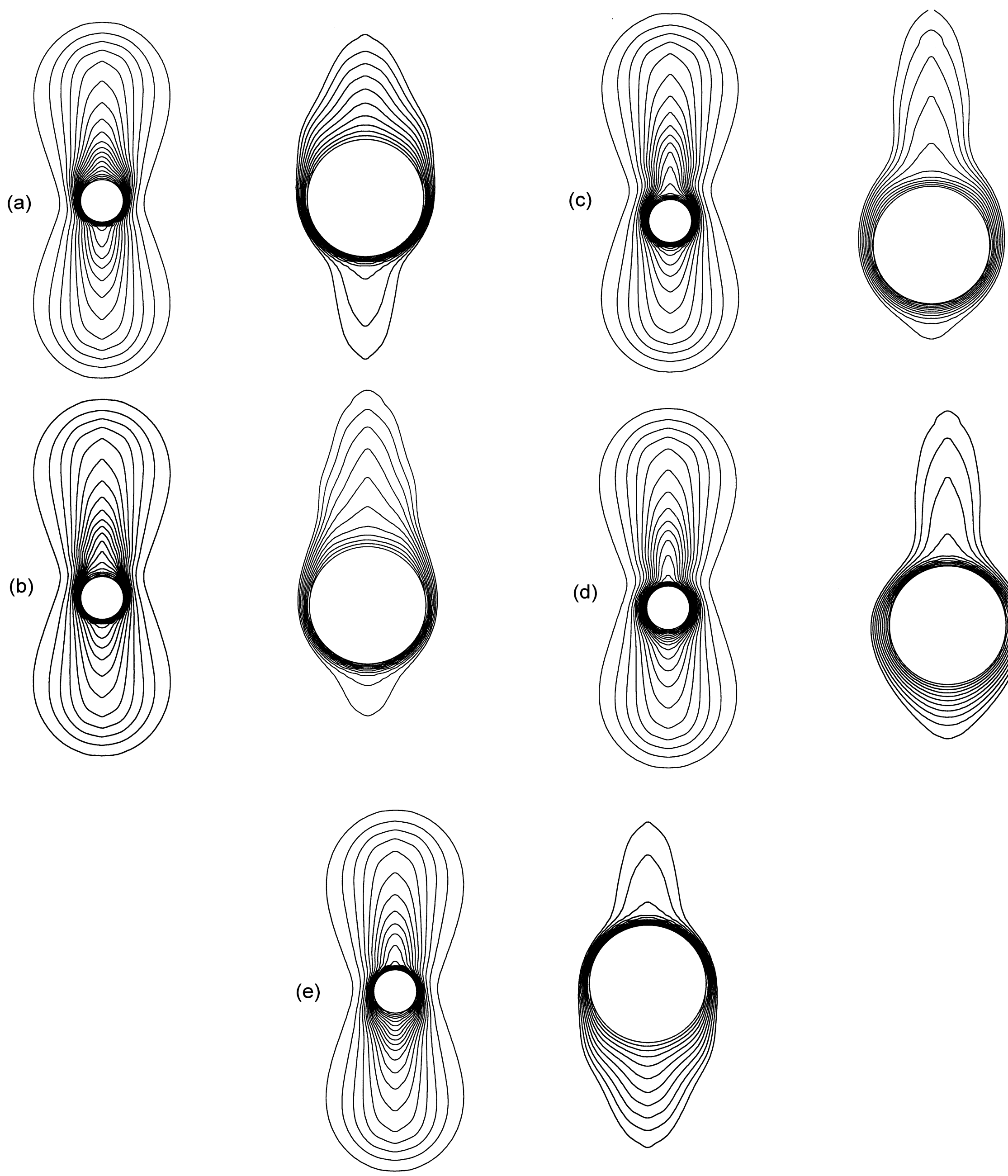

Fig. 10. The isotherm contours for the case of Re = 200, S = $\pi/4$. Isotherms plotted for left figures are 0.05, 0.1, …, 1.0. Isotherms plotted for magnified (right) figures are 0.5, 0.55, …, 1.0. (a) $\tau = 0.0$, (b) $\tau = 0.125$, (c) $\tau = 0.25$, (d) $\tau = 0.375$, (e) $\tau = 0.5$.

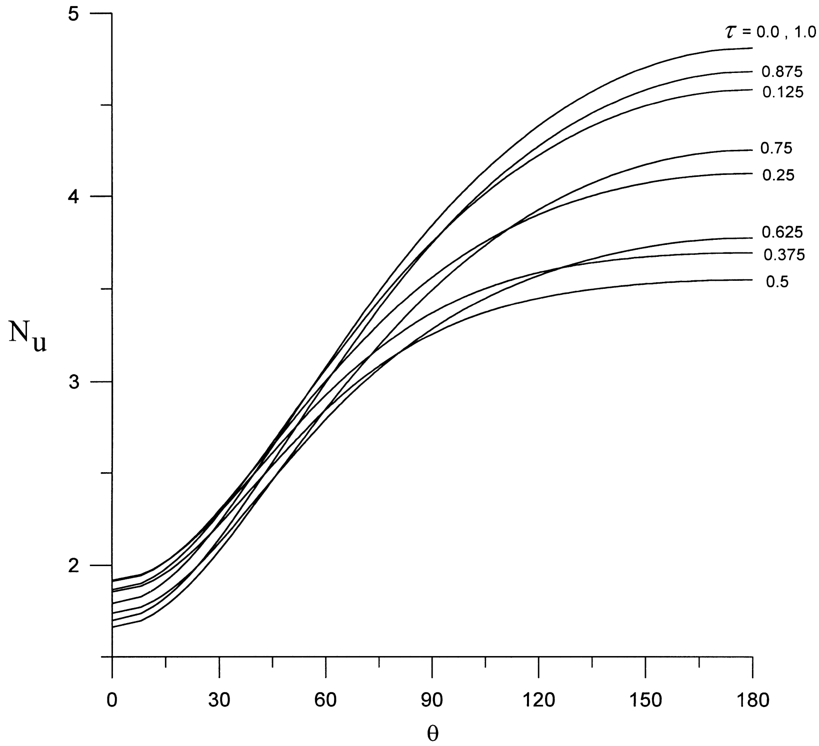

Fig. 11. The time variation of N_u distribution for the case of Re = 5, S = π/4, Gr = 250 during one complete cycle.

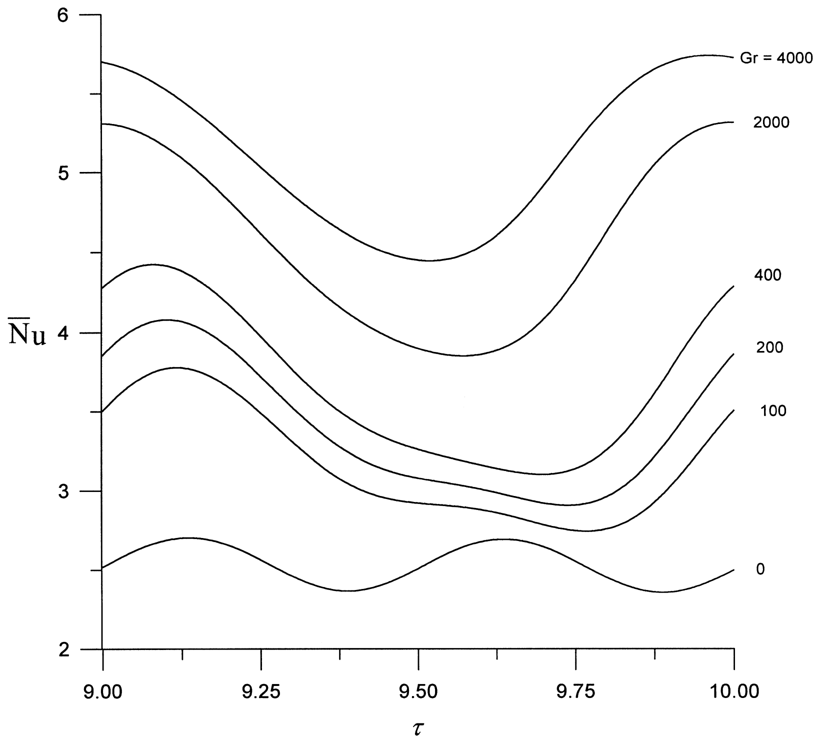

Fig. 12. The time variation of $\overline{N}_u$ for the case of Re = 20, S = π/4 at different Grashof numbers during one complete oscillation.

Table 3
The time (τ) at which the maximum and minimum $\overline{N}_u$ occur

Re	S	τ for $N_{u\,max}$	τ for $N_{u\,min}$
10	π/16	0.0775	0.3300
10	π/8	0.1100	0.3600
10	π/4	0.1325	0.3825
10	π/2	0.1500	0.4000
10	π	0.1650	0.4150
10	2π	0.1775	0.4275
5	π/4	0.1250	0.3775
20	π/4	0.1375	0.3875
50	π/4	0.1300	0.3825
100	π/4	0.0925	0.3575
200	π/4	0.0550	0.3375

Table 4
Values of $\overline{N_u}$ during cycles 4 and 20 at maximum positive and maximum negative (between brackets) free-stream velocities

Re	S	Cycle 4	Cycle 20
5	π/4	2.382740 (2.358426)	2.185313 (2.183641)
10	π/16	2.795186 (2.771123)	2.600988 (2.599335)
10	π/4	2.561382 (2.525726)	2.271497 (2.269053)
10	π	2.883390 (2.818388)	2.352940 (2.348442)
20	π/4	2.821154 (2.768285)	2.390656 (2.387051)
50	π/4	3.375515 (3.285642)	2.667035 (2.661329)
100	π/4	4.145011 (4.029703)	3.406128 (3.402050)
200	π/4	5.822037 (5.749165)	5.441513 (5.439799)

Table 5
Values of $\bar{\bar{N}}_u$ at different Grashof numbers

Re	Gr	Gr/Re^2	$\bar{\bar{N}}_u$	$\bar{\bar{N}}_u/\bar{\bar{N}}_u$ (forced)
5	0.0	0.0	2.2394	1.00
5	12.5	0.5	2.5586	1.14
5	25	1.0	2.6957	1.20
5	125	5.0	3.1186	1.39
5	250	10.0	3.3741	1.51
20	0.0	0.0	2.5296	1.00
20	100	0.25	3.1602	1.25
20	200	0.5	3.3862	1.34
20	400	1.0	3.6587	1.45
20	2000	5.0	4.5380	1.79
20	4000	10.0	5.0858	2.01

overall Nusselt number is found to fluctuate at one half of the frequency detected in the case of forced convection with a considerable deviation in the local Nusselt number distribution. Contrary to the case of steady uniform flow, a small increase of Gr/Re^2 may cause an appreciable increase in the overall heat transfer in oscillating flows.

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